Influence of Symmetry Energy Density on Isovector Giant Dipole Resonance

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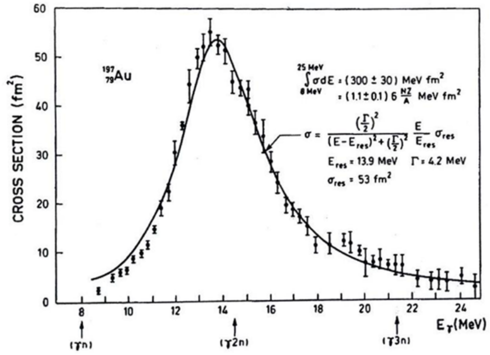
Isovector Giant Dipole Resonance



[8]

In the Isovector Giant Dipole Resonance (IVGDR), the nucleus remains spherical with the protons and neutrons oscillating internally antiphase to each other.

Motivation

- Properties of Giant Resonances (GR) can be extracted from experiment and used to investigate Nuclear Matter properties.
- Previous research established 3 constraints on NM properties but found no correlation between the centroid energy (E_{cen}) of the IVGDR and the symmetry energy coefficient (J) and its first, second, and third derivatives (L, K_{sym}, and Q_{sym} respectively) [1].
- The goal of this project is to narrow down the 33 interactions used in past research and determine if a correlation with J emerges.

Objectives

1. Narrow down the Skyrme interactions that satisfy established constraints on the incompressibility coeficcient (K_{NM} = 210-240 MeV), effective mass (m^*/m = 0.7-0.9), and enhancement coefficient (κ = 0.25-0.70) of the energy weighted sum rule of the IVGDR.
2. Calculate predictions of E_{cen} and α_D of the IVGDR for each interaction.
3. Determine correlations between the calculated results and each of the relevant NM properties (J, L, K_{sym}, Q_{sym}).
4. Constrain any NM properties associated with strong correlations using experimental values of E_{cen} and α_D.

Calculations of E_{cen} and α_D are carried out for ^{40,48}Ca, ^{90}Zr, ^{116}Sn, and ^{208}Pb nuclei.

Model

The nucleus is treated as a many-body quantum system with interactions due to the strong force denoted by V_{ij} . The hamiltonian is as follows:

$$H = \sum_i \frac{p_i^2}{2m} + \sum_{i<j} V_{ij} \,.$$

The large number of terms in $H\left|\psi\right\rangle = E\left|\psi\right\rangle$ makes solving for the ground state not feasible. To simplify the problem, each particle is treated as subject to a mean field, U.

Skyrme Interactions

The standard form of the Skyrme interaction we adopt is:

$$
\begin{aligned}
V_{ij} = t_0(1 + x_0 P_{ij}^{\sigma})\delta(\vec{r}_i - \vec{r}_j) + \tfrac{1}{2}t_1(1 + x_1 P_{ij}^{\sigma}) \\
\times \left[\overleftarrow{k}^2{}_{ij}\delta(\vec{r}_i - \vec{r}_j) + \delta(\vec{r}_i - \vec{r}_j)\vec{k}_{ij}^2\right] \\
+ t_2(1 + x_2 P_{ij}^{\sigma})\overleftarrow{k}^2{}_{ij}\delta(\vec{r}_i - \vec{r}_j)\vec{k}_{ij}^2 \\
+ \tfrac{1}{6}t_3(1 + x_3 P_{ij}^{\sigma})\rho^{\alpha}\left(\tfrac{\vec{r}_i + \vec{r}_j}{2}\right)\delta(\vec{r}_i - \vec{r}_j) \\
+ iW_0\overleftarrow{k}^2{}_{ij}\delta(\vec{r}_i - \vec{r}_j)(\vec{\sigma}_1 + \vec{\sigma}_2) \times \vec{k}_{ij}^2
\end{aligned}
$$

There are ten parameters that can be varied to give different Skyrme interactions.

Hartree-Fock Method

The wavefunction of the nucleus must be antisymmetric, as it is comprised of fermions. Here, ψ_i are the single particle wavefunctions.

$$\Psi(x_1, x_2, ..., x_N) = \frac{1}{\sqrt{N!}} \begin{vmatrix} \psi_1(x_1) & \cdots & \psi_1(x_N) \\ \vdots & \ddots & \vdots \\ \psi_N(x_1) & \cdots & \psi_N(x_N) \end{vmatrix} \quad .$$

The use of a Slater determinant enforces antisymmetry. The ground state of the wavefunction is determined by finding the single particle wavefunctions that minimize the ground state energy, $E = \langle \Psi \,| H |\, \Psi \rangle$, through the variational process ($\psi_i \rightarrow \psi_i + \delta\psi_i$), obtaining the coupled Hartree-Fock equations.

Hartree-Fock Method

$$-\frac{\hbar}{2m}\Delta\psi_i(\mathsf{r})+U_H(\mathsf{r})\psi_i(\mathsf{r})-\int U_F(\mathsf{r},\mathsf{r}')\psi_i(\mathsf{r}')d^3\mathsf{r}' = e_i\psi_i(\mathsf{r}),\;\; i=1,\ldots,A$$

$$\text{where, } U_H(\mathsf{r},\mathsf{r}') = \sum_{k=1}^{A}\int \psi_k^*(\mathsf{r}')V(\mathsf{r},\mathsf{r}')\psi_k(\mathsf{r}')d^3\mathsf{r}'$$

$$U_F(\mathsf{r},\mathsf{r}') = \sum_{k=1}^{A}\psi_k^*(\mathsf{r}')V(\mathsf{r},\mathsf{r}')\psi_k(\mathsf{r}')$$

Here, $U_H(\mathsf{r})$ and $U_F(\mathsf{r})$ are the direct and exchange potentials given in terms of the two-body interaction and the single particle wave functions. The Hartree-Fock equations are solved iteratively using the wavefunction of a particle within a Woods-Saxon potential as an initial guess.

Random Phase Approximation

RPA is used to calculate the excited states, $|j\rangle$. The strength function $S(E)$ is obtained using the scattering operator F_{LM}, where $f(r_n) = r$ and $L = 1$ for the IVGDR.

$$S(E) = \sum_j \left|\langle 0 \left| F_{LM} \right| j \rangle\right|^2 \delta(E_j - E_0)$$

$$F_{LM} = \tfrac{Z}{A} \sum_n f(r_n) Y_{LM}(n) - \tfrac{N}{A} \sum_p f(r_p) Y_{LM}(p)$$

The moments of the strength function are calculated and used to determine E_{cen} and α_D.

$$m_k = \int_{E_1}^{E_2} E^k S(E) dE \,, \quad E_{cen} = \frac{m_1}{m_0} \,, \quad \alpha_D = \frac{24\pi e^2}{9} m_{-1}$$

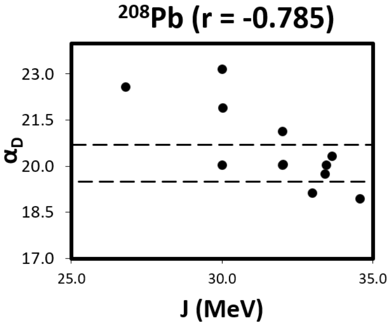
NM Properties

The NM properties we are concerned with are defined by the following equations, where ρ_0 is the saturation density, E_0 is the ground state energy, and E_{sym} is the symmetry energy.

$$K_{NM} = 9\rho_0^2 \frac{\partial^2 E_0}{\partial \rho^2}|_{\rho_0} , \quad J = E_{sym}[\rho_0] , \quad L = 3\rho_0 \frac{\partial E_{sym}}{\partial \rho}|_{\rho_0}$$

$$K_{sym} = 9\rho_0^2 \frac{\partial^2 E_{sym}}{\partial \rho^2}|_{\rho_0} , \quad Q_{sym} = 27\rho_0^3 \frac{\partial^3 E_{sym}}{\partial \rho^3}|_{\rho_0}$$

CORRELATIONS AND GRAPHS



Each interaction predicts different values of NM properties, E_{cen}, and α_D.

The linear correlation coefficient and experimental range, represented as between the dashed lines, are shown [2]-[7].

Pearson Correlation Coefficient:

$$r_{X,Y} = \frac{cov(X,Y)}{\sigma_X \sigma_Y} = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_i (x_i - \bar{x})^2}\sqrt{\sum_i (y_i - \bar{y})^2}}$$

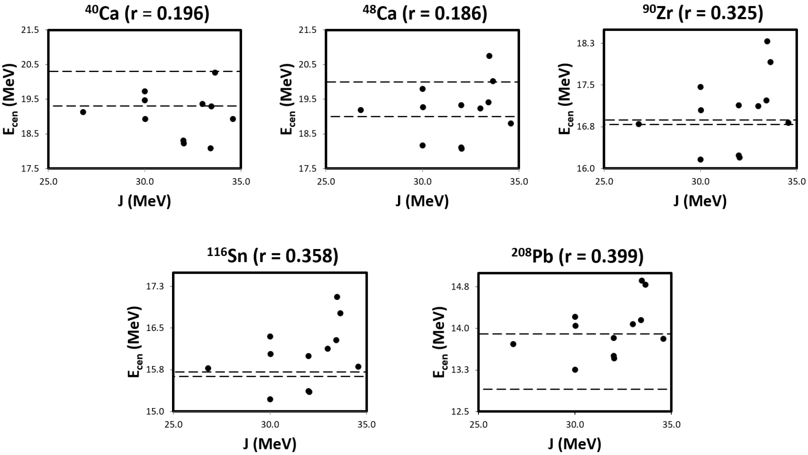
Correlation Coefficients

Linear correlation coefficients are calculated for each of the 5 nuclei, with the average correlations shown below. The correlations seen with the set of interactions used in prior research are shown for comparison.

Mean Correlations Between NM properties and E_{cen} and α_D for the IVGDR

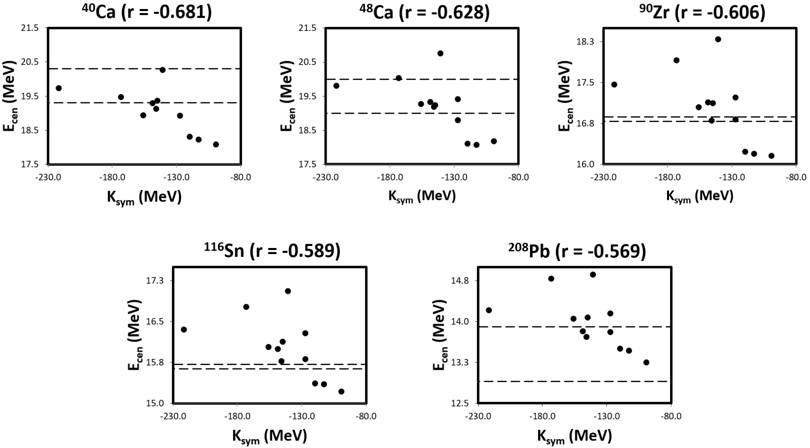
	K_{NM}	J	L	K_{sym}	Q_{sym}	m^*/m	κ
33 Interactions (E_{cen})	0.00	-0.34	-0.40	-0.30	0.41	-0.60	0.84
12 Interactions (E_{cen})	-0.32	0.29	0.17	-0.62	-0.23	0.32	0.64
33 Interactions (α_D)	-0.14	0.27	0.52	0.43	-0.65	0.20	-0.14
12 Interactions (α_D)	-0.14	-0.66	-0.09	0.31	-0.50	0.07	0.30

J and E_{cen}

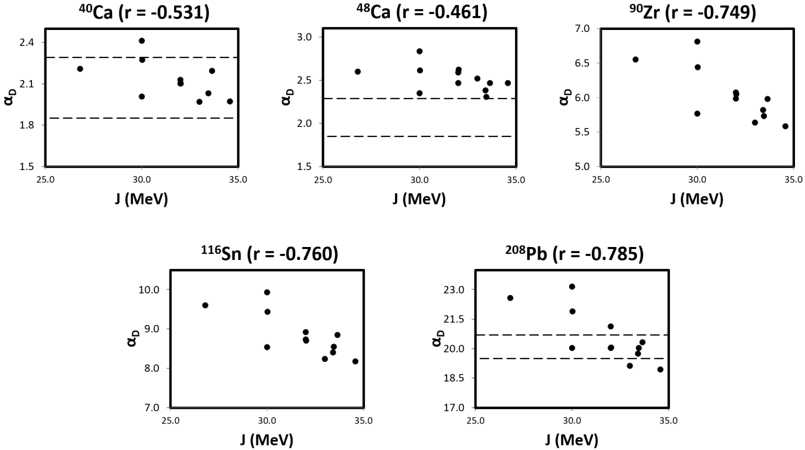


^{40}Ca (r = 0.196)
^{48}Ca (r = 0.186)
^{90}Zr (r = 0.325)
^{116}Sn (r = 0.358)
^{208}Pb (r = 0.399)
E_{cen} (MeV)
J (MeV)

K_{sym} AND E_{cen}



J and Electric Polarizability



Conclusions

- No correlation between E_{cen} and J was revealed.
- Low to medium correlations were seen between α_D and J as well as between E_{cen} and K_{sym}.
- No correlations were large enough to justify imposing NM property constraints.
- The same lack of influence of J on the IVGDR seen in past research is seen here.

References

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Questions?